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In this work we consider the discrete time detection of strong mixing signals in strong mixing noise, and we allow a large degree of dependency to exist between the signal and the noise. We investigate the memoryless detector which is optimum in the sense of the asymptotic relative efficiency. It is shown that the design of this detector reduces to the solution of an integral equation in which knowledge of only the second-order statistics of the random processes involved is required.

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SOME RESULTS ON ASYMPTOTIC MEMORYLESS DETECTION IN STRONG MIXING NOISE

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Abstract

In this work we consider the discrete time detection of strong mixing signals in strong mixing noise, and we allow a large degree of dependency to exist between the signal and the noise. We investigate the memoryless detector which is optimum in the sense of the asymptotic relative efficiency. It is shown that the design of this detector reduces to the solution of an integral equation in which knowledge of only the second-order statistics of the random processes involved is required.

I. INTRODUCTION

The detection of signals in corrupting noise has been an area of interest for some time. Because of modern high speed sampling, it is expected that the underlying random processes involved will not be "white", but will instead possess dependency to a certain degree. Neyman-Pearson optimal techniques [1] are tractable only in cases where the appropriate multivariate distributions are known. In many non-Gaussian situations these distributions are not known, which has thus led to the choice of an alternate fidelity criterion, commonly the asymptotic relative efficiency (ARE) criterion, which is especially appropriate in the weak signal and large sample situation. Because continuous time detection is often intractable in the non-Gaussian case, current efforts are directed toward discrete time detection. Results in this area have been obtained recently by Poor and Thomas [2,3] for the case of memoryless detection of a known constant signal in additive m-dependent noise; we have shown [4,5] how these results may be extended to a large class of ϕ-mixing noises. Because an assumption of a constant signal is in many cases overly restrictive, we have also considered [6] the case where both the signal and noise may be modeled as ϕ-mixing processes, where a large degree of dependency may also occur between the signal and noise. All of these results have the advantage of requiring only second-order statistical knowledge of the random processes involved.

The employment of the ϕ-mixing models of [4-6] is motivated by an expectation that dependency between samples gradually "decreases" as the samples are more widely separated in time, and the formal definition of a ϕ-mixing process is consistent with such a property. The class of ϕ-mixing processes employed may be seen to be quite general, however as described in the next section, the formal definition is more restrictive than what we might expect as the most natural consequence of our intuition. In this paper we will model the signal and noise in a way which is in many ways much more consistent with our intuition. This will be achieved through the employment of strong mixing processes to model the signal and noise. The class of strong mixing processes is more general than that of ϕ-mixing processes, and because of ties to the maximal correlation coefficient the validity of a strong mixing model is easier to check. We therefore will consider the general situation where we are detecting the presence of a strong mixing signal in strong mixing noise.

II. PRELIMINARIES

Let $\{X_i;\ i=1,2,\ldots\}$ be a strictly stationary sequence of random variables. For $a \le b$, define $M(a,b) = \sigma\{X_a, X_{a+1}, \ldots, X_b\}$, the σ-algebra generated by the indicated random variables, where a and b may take on extended real values. Then $\{X_i;\ i=1,2,\ldots\}$ is <u>symmetrically ϕ-mixing</u> if there exists a nonnegative sequence $\{\phi_i;\ i=1,2,\ldots\}$ with $\phi_i \to 0$ such that for each k, $1 \le k < \infty$, and for each $i \ge 1$, $E_1 \in M(1,k)$ and $E_2 \in M(k+i,\infty)$ together imply

$$|P(E_1 \cap E_2) - P(E_1)P(E_2)| \le \phi_i \min\{P(E_1), P(E_2)\}.$$

In [4-6] the above type of process is employed. Note that the left side of the above inequality provides a measure of dependence between events E_1 and E_2, and the right side bounds this quantity with a term involving E_1 and E_2. Such a definition has computational advantages; for example, it results in the very powerful Lemma 1 of [7, p.170]. However, it is a stronger requirement than our intuition might demand. Since we really wish to simply require a "decrease" in dependency as E_1

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and E_2 are more widely separated in time, it is thus more natural to employ the weaker requirement that there exists a nonnegative sequence $\{\alpha_i;\ i=1,2,\ldots\}$ with $\alpha_i \to 0$ such that for all E_1 and E_2 as above we have

$$|P(E_1 \cap E_2) - P(E_1)P(E_2)| \leq \alpha_i.$$

A process satisfying this condition is called <u>strong mixing</u>. We will consider the detection of a strong mixing signal $\{S_i;\ i=1,2,\ldots\}$, where $0 < E\{S_1^2\} < \infty$, in additive strong mixing noise $\{N_i;\ i=1,2,\ldots\}$, where we observe realizations $\{y_i;\ i=1,2,\ldots,n\}$ of the process $\{Y_i; i=1,2,\ldots,n\}$. In order to apply the ARE fidelity criterion, this will amount to a choice between the two hypotheses

$$H_0:\ Y_i = N_i;\ i=1,2,\ldots,n$$

$$H_1:\ Y_i = N_i + \theta S_i;\ i=1,2,\ldots,n$$

where θ is a parameter which will be allowed to approach zero at the proper rate, thus yielding the asymptotic limit. Throughout the discussion we will assume that both the noise and signal processes possess (possibly different) α-representations which satisfy

$$\sum_{i=1}^{\infty} \alpha_i^{\ \delta/(2+\delta)} < \infty$$

for some appropriate $\delta > 0$. Such a strong mixing process will be called <u>δ-acceptable</u>. For convenience we assume the existence of densities $f_j(\cdot,\cdot)$ of N_k and N_{k+j}, $f(\cdot)$ of N_1, $f(\cdot,\cdot)$ of N_k and S_k, where the latter is assumed to be independent of k. We also assume

$$K_n(x,y) \triangleq \sum_{j=1}^{n} [f_j(x,y) + f_j(y,x)]/\sqrt{f(x)f(y)}$$

is square integrable for all n, and that $f(\cdot)$ is strictly positive on the real line. We assume in addition that

$$\int y^2 \frac{\partial^2}{\partial x^2} f(x,y)dy/\sqrt{f(x)}$$

and

$$\int y \frac{\partial}{\partial x} f(x,y)dy/\sqrt{f(x)}$$

are square integrable. Note that if the signal and noise are independent, the latter condition is equivalent to the assumption of finite Fisher's information number contained in [2,3] and [4,5]. We also assume that

$$\lim_{\theta \to 0} \int f(x-\theta y,y)dy = f(x).$$

As in [2-6], we will optimize over the class of optimal memoryless detectors designed under a "white noise" assumption, i.e. where a test statistic $T_g(y) = \sum_{i=1}^{n} g(y_i)$ is compared to a threshold. Specifying g will therefore be of prime concern.

We will restrict the class $\mathscr{G}$ of nonlinearities g to include those measurable real valued functions for which we can find $\theta_1 > 0$ and $\delta_1 > \delta$ such that the random variable $g(N_1+\theta S_1)$ satisfies $E\{|g(N_1+\theta S_1)|^{2+\delta_1}\} < \infty$ for all $\theta \in [0,\theta_1]$, and such that the following mild regularity conditions hold, where $E_\theta(\cdot)$ denotes expectation computed under H_1 with parameter θ (by proper choice of the threshold, we assume without loss of generality that the random variables $g(N_i)$ are zero mean):

(a) $\int g(x)f'(x)dx \neq 0$

if the signal and noise processes are independent;

(b) $$\lim_{n\to\infty} \frac{\left[\frac{\partial}{\partial\theta} E_\theta\{T_g(Y)\}|_{\theta=0}\right]^2}{nE_0\{[T_g(Y)]^2\}} \triangleq \eta(g) > 0$$

if $\iint yg(x) \frac{\partial}{\partial x} f(x,y)dxdy \neq 0$, or

(b') $$\lim_{n\to\infty} \frac{\left[\frac{\partial^2}{\partial\theta^2} E_\theta\{T_g(Y)\}|_{\theta=0}\right]^2}{nE_0\{[T_g(Y)]^2\}} \triangleq \eta(g) > 0$$

if $\iint yg(x) \frac{\partial}{\partial x} f(x,y)dxdy = 0$;

(c) $$-\infty < \lim_{n\to\infty} \frac{\partial}{\partial\theta} E\{g(N_1+\theta S_1)\}|_{\theta=k_1/\sqrt{n}} = \frac{\partial}{\partial\theta} E\{g(N_1+\theta S_1)\}|_{\theta=0} < \infty$$

for some constant $k_1 > 0$

if $\iint yg(x) \frac{\partial}{\partial x} f(x,y)dxdy \neq 0$, or

(c') $$-\infty < \lim_{n\to\infty} \frac{\partial^2}{\partial\theta^2} E\{g(N_1+\theta S_1)\}|_{\theta=k_2/n^{1/4}} = \frac{\partial^2}{\partial\theta^2} E\{g(N_1+\theta S_1)\}|_{\theta=0} < \infty$$

for some constant $k_2 > 0$

if $\iint yg(x) \frac{\partial}{\partial x} f(x,y)dxdy = 0$;

(d) $$\lim_{\theta\to 0} E\{g(N_1+\theta S_1)^{2+\delta}\} = E\{g(N_1)^{2+\delta}\};$$

(e) $$\frac{\partial}{\partial\theta} \iint g(x)f(x-\theta y,y)dxdy|_{\theta=0} = \iint \frac{\partial}{\partial\theta} g(x)f(x-\theta y,y)|_{\theta=0}\, dxdy$$

if $\iint yg(x) \frac{\partial}{\partial x} f(x,y)dxdy \neq 0$, or

(e') $\frac{\partial^2}{\partial\theta^2} \iint g(x)f(x-\theta y,y)dxdy|_{\theta=0}$

$$= \iint \frac{\partial^2}{\partial\theta^2} g(x)f(x-\theta y,y)|_{\theta=0}\, dxdy$$

if $\iint yg(x)\frac{\partial}{\partial x} f(x,y)dxdy = 0$;

(f) $\sigma_0^2(g) \triangleq E\{g(N_1)^2\} + 2\sum_{j=1}^{\infty} E\{g(N_1)g(N_{j+1})\} > 0.$

Properties (a)-(c') are assumptions conventionally imposed for application of the Pitman-Noether theorem [8], whereas (d)-(e') are exceedingly mild restrictions. For a large class of processes, including all of the examples of [3], property (f) is satisfied and may therefore be ignored. Note that these assumptions are the analog to those of [6].

III. DEVELOPMENT

The great appeal of the asymptotic fidelity criterion employed is that it allows appeal to central limit theorem results. We first need the following lemma, which is the analog to Lemma 1 of [6]:

Lemma 1: If $\{N_i;\ i=1,2,\ldots\}$ and $\{S_i;\ i=1,2,\ldots\}$ are δ-acceptable and independent processes, then the process $N_1,S_1,N_2,S_2,\ldots$ is δ-acceptable.

Proof: We may repeat the first part of the proof of Lemma 1 of [6] to obtain

$$|E\{I_{E_1}h_j\} - E\{I_{E_1}\}E\{h_j\}|$$

$$\leq |\int\{\int \tilde{w}\tilde{h}_j dF_N(k,j) - \int \tilde{w}dF_N \int \tilde{h}_j dF_N(j)\}\, dF_S(k,j)|$$

$$+ |\iint \tilde{w}dF_N \int \tilde{h}_j dF_N(j)dF_S(k,j)$$

$$- \iiiint \tilde{w}\tilde{h}_j dF_N dF_N(j) dF_S dF_S(j)|,$$

where we employ the notation of [6]. Application of Lemma 1.2 of [9] together with the fact that $|\tilde{w}| \leq 1$ and $|\tilde{h}_j| \leq 1$ a.s. then shows that first summand on the right side of the inequality can be upper bounded by $4\alpha_i$, where $\{\alpha_i;\ i=1,2,\ldots\}$ is an α-representation for $\{N_i; i=1,2,\ldots\}$. In a similar manner the second summand can be upper bounded by $4\beta_i$, where $\{\beta_i;\ i=1,2,\ldots\}$ is an α-representation for $\{S_i;\ i=1,2,\ldots\}$. We therefore obtain

$$|E\{I_{E_1}h_j\} - E\{I_{E_1}\}E\{h_j\}| \leq 4(\alpha_i+\beta_i)$$

and the desired result follows from the proof of Lemma 1 of [6]. Q.E.D.

When dependency is present between the two processes, we can obtain an analogous result to Lemma 1 if the noise is dependent on a finite "window" of the signal, such as the signal-dependent noise induced through reverberation effects. The extension of Lemma 1 to this signal-dependent case is given by the following:

Lemma 2: Suppose $\{S_i;\ i=1,2,\ldots\}$ is δ-acceptable, and for a fixed nonnegative integer m, $N_i = G(X_i,Z_i)$ for $i=1,2,\ldots$ where X_i is $\sigma\{S_{i-m},\ldots,S_{i+m}\}$ measurable, $G\colon \mathbb{R}^2 \to \mathbb{R}$ is measurable, and $\{Z_i;\ i=1,2,\ldots\}$ is δ-acceptable and independent of $\{S_i;\ i=1,2,\ldots\}$ (we let $S_{i-m} \triangleq S_1$ for $i \leq m$). Then $N_1,S_1,N_2,S_2,\ldots$ is δ-acceptable.

Proof: This follows through an argument identical to the proof of Lemma 2 of [6]. Q.E.D.

We can thus obtain a useful result:

Theorem 1: Suppose $\{\mathscr{F}_i\colon \mathbb{R}^{2p+2} \to \mathbb{R};\ i=1,2,\ldots\}$ is a family of measurable functions where p is a fixed nonnegative integer. Then under the hypothesis of Lemma 1 or Lemma 2, we have that $\{\mathscr{F}_i(N_i,S_i,\ldots,N_{i+p},S_{i+p});\ i=1,2,\ldots\}$ is δ-acceptable.

Proof: This follows as a consequence of Lemma 1, Lemma 2, and a straightforward modification of Proposition 7 of [5]. Q.E.D.

We can now obtain the result which will allow employment of the Pitman-Noether Theorem [8].

Theorem 2: Suppose $\theta_n \in \mathbb{R}$ with $\theta_n \to 0$, and $g \in \mathscr{G}$. Let $T_{n,\theta} = T_g/\sqrt{n}$ under H_1 with parameter θ_n, where the noise and signal processes satisfy the hypothesis of Lemma 1 or Lemma 2. If $\sigma^2_{n,\theta} \triangleq E\{(T_{n,\theta} - E\{T_{n,\theta}\})^2\}$, then

$$\frac{T_{n,\theta} - E\{T_{n,\theta}\}}{\sigma_{n,\theta}} \xrightarrow{\mathscr{L}} N(0,1).$$

Proof: Letting $T_{n,0} = T_g/\sqrt{n}$ under H_0 it follows from condition (f) that $\lim_{n\to\infty} \sigma^2_{n,0} = \sigma^2_0(g) > 0$, and hence, $\lim_{n\to\infty} n\sigma^2_{n,0} = \infty$. We thus obtain from Theorem 1.4 of [9] and Theorem 1 that

$$\frac{\sqrt{n}\,T_{n,0}}{\sqrt{n}\,\sigma_{n,0}} \xrightarrow{\mathscr{L}} N(0,1),$$ and therefore

$T_{n,0}/\sigma_{n,0} \xrightarrow{\mathscr{L}} N(0,1)$. We also have from Lemma 1.3 of [9] and Theorem 1 that

$$E\{(T_{n,\theta} - E\{T_{n,\theta}\} - T_{n,0})^2\}^{\frac{1}{2}}$$

$$\leq E\{(g(N_1+\theta_n S_1) - g(N_1))^2\}^{\frac{1}{2}}$$

$$+2[(4+6E\{(g(N_1+\theta_n S_1)-g(N_1))^{2+\delta}\}) \sum_{i=1}^{\infty} \gamma_i^{\delta/(2+\delta)}]^{\frac{1}{2}}$$

$$+ |E\{g(N_1+\theta_n S_1)\}|,$$

where $\sum_{i=1}^{\infty} \gamma_i^{\delta/(2+\delta)} < \infty$ and $\{\gamma_i;\ i=1,2,\ldots\}$ is an α-representation for the δ-acceptable process

$\{g(N_i+\theta_n S_i)-g(N_i);\ i=1,2,\ldots\}$. Using a result from [10], we conclude from assumptions (c)-(c') and (d) that $E\{(T_{n,\theta}-E\{T_{n,\theta}\}-T_{n,0})^2\}\to 0$, which yields the desired result from [7, p. 25]. Q.E.D.

We can now obtain the main result:

Theorem 3: Suppose that the hypothesis of Lemma 1 or Lemma 2 is satisfied, and $g \in \mathscr{G}$. Then g is optimal (in the sense of the ARE) if and only if g satisfies (up to a scale factor)

$$\text{(A)}\quad \sum_{j=1}^{\infty}\int[f_j(x,y)+f_j(y,x)]g(y)dy+f'(x) = -f(x)g(x)$$

if $\{N_i\}_{i=1}^{\infty}$ and $\{S_i\}_{i=1}^{\infty}$ are independent and $E\{S_1\}\neq 0$, or

$$\text{(B)}\quad \sum_{j=1}^{\infty}\int[f_j(x,y)+f_j(y,x)]g(y)dy+f''(x) = -f(x)g(x)$$

if $\{N_i;\ i=1,2,\ldots\}$ and $\{S_i;\ i=1,2,\ldots\}$ are independent and $E\{S_1\}=0$, where $f_j(\cdot,\cdot)$ is the joint density of N_1 and N_{j+1}, and f is the *univariate density of N_1, or*

$$\text{(C)}\quad \sum_{j=1}^{\infty}\int[f_j(x,y)+f_j(y,x)]g(y)dy+\int y\frac{\partial}{\partial x}f(x,y)dy = -f(x)g(x),$$

if $\iint yg(x)\frac{\partial}{\partial x}f(x,y)dxdy\neq 0$, or

$$\text{(D)}\quad \sum_{j=1}^{\infty}\int[f_j(x,y)+f_j(y,x)]g(y)dy+\int y^2\frac{\partial^2}{\partial x^2}f(x,y)dy = -f(x)g(x),$$

if $\iint yg(x)\frac{\partial}{\partial x}f(x,y)dxdy = 0$, where $f(\cdot,\cdot)$ is the joint density of N_1 and S_1.

Proof: In view of Theorem 2, the proof of Theorem 3 of [6] may be repeated. Q.E.D.

Because of the assumption of the existence of a signal density, the above results do not directly apply for the case of a constant signal. However a similar modification of the methods of [4,5] may be employed to show that the appropriate condition for the nonlinearity g in the constant signal case is given by equation (A) of Theorem 3. Thus as in [6] we note that if the signal and noise are independent, the optimal nonlinearity for the case of a random signal with nonzero mean is the same as that for the constant signal case.

As in [4-6], the above integral equations are of nonstandard form. If we note that each of the equations is of form

$$\sum_{j=1}^{\infty}\int[f_j(x,y)+f_j(y,x)]g(y)dy+h(x) = -f(x)g(x),$$

then, as in [4-6], we might wish to approximate the nonlinearity g by nonlinearities g_m which are solutions to the equations

$$\sum_{j=1}^{m}\int[f_j(x,y)+f_j(y,x)]g_m(y)dy+h(x) = -f(x)g_m(x).$$

In this case we would then hope that the convergence in some appropriate sense of the g_m would lead to an optimal nonlinearity. This question is answered in the following:

Theorem 4: Under the hypothesis of Lemma 1 or Lemma 2, if there exists a $g\in\mathscr{G}$ such that a subsequence $\{g_{m_k}\}_{k=1}^{\infty}$ of $\{g_m\}_{m=1}^{\infty}$ satisfies $g_{m_k}(N_1)-g(N_1)\to 0$ in $L_{2+\delta_1}$, then g is optimal.

Proof: We may repeat the first part of the proof of Proposition 3 of [5], and conclude that it is sufficient to show

$$\sum_{j=1}^{m_k}\iint[f_j(x,y)+f_j(y,x)][g_{m_k}(y)-g(y)]\tilde{\delta}g(x)dxdy + \sum_{j=m_k+1}^{\infty}\iint[f_j(x,y)+f_j(y,x)]g(y)\tilde{\delta}g(x)dxdy\to 0$$

as $k\to\infty$, where $\tilde{\delta}g$ is an arbitrary zero mean variation satisfying $E\{|\tilde{\delta}g(N_1)|^{2+\delta_1}\}<\infty$, where $\delta_1>\delta$. Application of Lemma 1.3 of [9] to the second summand above shows that it can be upper bounded by

$$[4+2(C_1+C_2+\sqrt{C_1C_2})]\sum_{j=m_k+1}^{\infty}\alpha_j^{\delta/(2+\delta)}\to 0,$$

where $C_1 = E\{|g(N_1)|^{2+\delta}\}<\infty$ and $C_2 = E\{|\tilde{\delta}g(N_1)|^{2+\delta}\}<\infty$. Moreover, a similar application together with the Schwarz inequality shows that the first summand can be upper bounded by

$$[4+2(D_1+D_2+\sqrt{D_1D_2})]^{1/(1+\varepsilon)}\sum_{j=1}^{\infty}\alpha_j^{\delta_1/[(2+\delta_1)(1+\varepsilon)]}\cdot E\{[\tilde{\delta}g(N_1)]^2\}^{\varepsilon/2}E\{[g_{m_k}(N_1)-g(N_1)]^2\}^{\varepsilon/2}$$

for any $\varepsilon>0$, where

$$D_1 = E\{|g_{m_k}(N_1)-g(N_1)|^{2+\delta_1}\}<\infty$$

$$D_2 = E\{|\tilde{\delta}g(N_1)|^{2+\delta_1}\}<\infty.$$

Choosing ε small enough so that

$\delta_1/[(2+\delta_1)(1+\varepsilon)]\geq\delta/(2+\delta)$,

we obtain the desired result. Q.E.D.

Note the conditions on the optimal nonlinearity g and the α-representation are exceedingly mild. For example, these results hold if

$E\{[g(N_1+\theta S_1)]^4\} < \infty$ for all $\theta \in [0,\theta_1]$,

$\sum_{i=1}^{\infty} \alpha_i^{1/3} < \infty$, and the subsequence of Theorem 4 converges in L_4. We remark finally that the nonlinearities g_m are obtainable through standard Hilbert-Schmidt techniques as solutions of Fredholm integral equations of the second kind.

IV. CONCLUSION

We have considered the design of the optimal detector for signal detection in corrupting noise, where both the signal and noise may be chosen from a large class of strong mixing processes and may be dependent on each other. We have seen that this design reduces to the solution of an integral equation in which knowledge of only the second-order statistics of the random processes involved is required. In particular, if the signal is independent of the noise and has nonzero mean, the optimal detector is the same as in the constant known signal case.

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BIOGRAPHIES

D. R. Halverson was born in Menomonie, Wisconsin on March 1, 1947. He received the B.A. degree in mathematical sciences summa cum laude from the University of Iowa in 1968, the A.M. degree in mathematics from the University of Illinois in 1972, and the Ph.D. degree in electrical engineering from the University of Texas at Austin in 1979. He was employed as a Mathematical Statistician with the U.S. Air Force at Kelly Air Force Base from 1972-75 and as a Mathematics Instructor at the U.S. Air Force Academy Preparatory School from 1975-77. He is presently with the Department of Electrical Engineering at Texas A&M University. His research interests include statistical communication theory, detection theory, and estimation theory. He is a member of Phi Beta Kappa and IEEE.

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